Syllabus for the Master Degree Programme in Mathematics

This syllabus describes the essential mathematical contents of the basic courses of the Master Programme in Mathematics. While it is understood that instructors will retain their teaching autonomy, it is also important that students may have a clear idea of the necessary prerequisites and of the natural sequence of the various topics.

Advanced Analysis A
Foundations of Banach and Hilbert spaces.

Basic functional analysis on normed spaces. Hahn-Banach, Banach-Steinhaus, Baire, open map, inverse map, closed graph theorems. Weak topology and weak* topology. Spaces of continuous functions: Stone-Weierstrass and Ascoli-Arzelà theorems. Hilbert spaces: general theory. Lax-Milgram theorem, continuous linear operators on Banach and Hilbert spaces.

Advanced Analysis B
Distribution theory and spaces of functions.

Functions of bounded variation and absolutely continuous functions. Distributions, temperate distributions: Fourier transforms. Sobolev spaces: embedding theorems, Sobolev and Poincaré inequalities.

Functional Analysis
Advances topics in functional analysis.

Spectral theory of selfadjoint compact operators: Fredholm alternative. Dofferential calculus in normed spaces: implicit function theorem. Topological degree: Brower, Leray-Schauder.

Advanced Geometry 1
Algebraic topology and singular homology.

Simplicial complexes and simplicial homology. Elements of homological algebra. Singular homology, homology with coefficients. CW complexes and cellular homology. Classification of surfaces. Cohomology and Poincaré duality.

Advanced Geometry 2
Differential varieties and differential forms.

Differential varieties and differentiable functions between varieties. Tangent and cotangent spaces, vector bundles. Vector fields. Differential forms and de Rham cohomology. Integration on varieties and Stokes theorem. Outline of Riemannian varieties.

Advanced Geometry 3
Algebraic geometry of affine and projective varieties.

Zariski topology on affine and projective varieties. Hilbert Nullstellensatz. Regular and rational maps. Tangent spaces and singular points. Blow-up and outline of the resolution of singularities. Selected topics in commutative algebra (commutative rings and modules).

Advanced algebra

Advanced course on topics in commutative algebra, or group theory (also group representations), or field theory.

Advanced Mathematical Physics A
Differential equations of Mathematics Physics.

Linear differential operators. Cauchy problem for PDE (partial differential equations), Cauchy-Kovalevskaya theorem. Wave, Laplace and heat equations.

Advanced Mathematical Physics B
Analytical mechanics.

Reminder on Newtonian Mechanics (mechanics of particle systems). Reminder on Lagrangian Mechanics. Hamiltonian Mechanics. Hamilton and Hamilton-Jacobi equations, Arnold's theorem. Symplectic varieties. Symmetries and Noether's theorem.